Physics

By

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Class:10+1

Unit:X

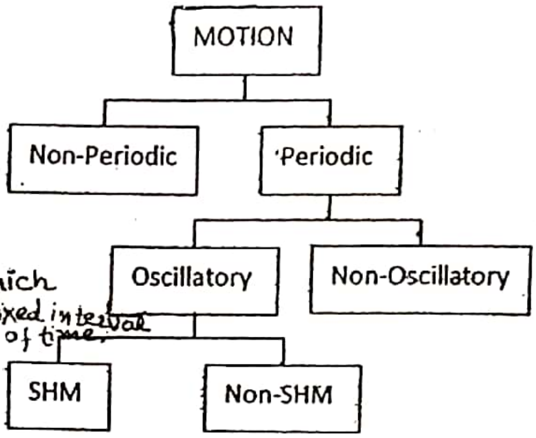
Topic: Oscillations and Waves

Unit X: Oscillations and Waves

Periodic motion - time period, frequency, displacement as a function of time. Periodic functions. Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a spring-restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period. Free, forced and damped oscillations (qualitative ideas only), resonance. Wave motion. Transverse and longitudinal waves, speed of wave motion. Displacement relation for a progressive wave. Principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect.

Q.1. Discuss following with examples:
a) Periodic, non-periodic motion
b) Oscillatory, non-oscillatory motion.
c) SHM, non-SHM

Ans.



a) **Periodic Motion:**

Periodic motion of a body is that motion in which phenomenon repeats itself after fixed interval of time.

Ex 1. The revolution of Earth around the Sun.

Ex 2. Rotation of Earth about its own axis.

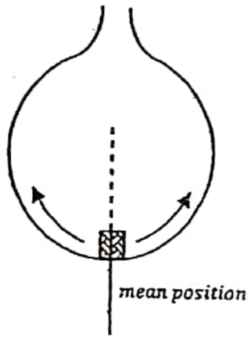
Non-Periodic:

Motion which is not repeated after a fixed interval of time.

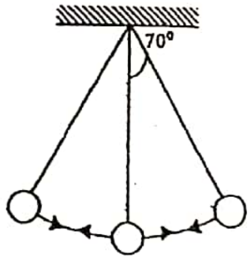
b) **Oscillatory Motion:**

Oscillatory or Vibrating motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point (mean position) in a definite interval of time.

Ex 1. Small marble left inside a pot. It rolls to and fro about its mean position.



Ex 2. Pendulum moving to and fro.



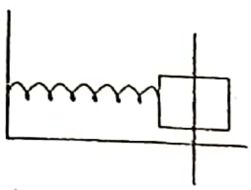
c) **SHM:**

It is oscillatory motion in which

$$\boxed{\text{Acc.} = -(\text{Const})\ \text{displacement}}$$

is valid, is called *SHM*.

Ex 1. For spring mass arrangement



$$F = -kx$$

$$Ma = -kx$$

$$a = -\left(\frac{k}{m}\right)x$$

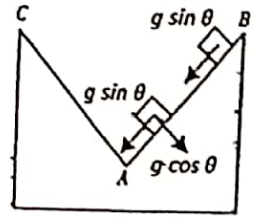
$$\text{Acc} = -(\text{Const})\ \text{displacement}$$

Non-SHM:

Motion in which acc = -(Const) displacement is not valid, is called *Non-SHM*.

11 / Ch 10 / Unit 10A / Q1 /
Periodic Oscillatory And SHM

Q2.a) A particle oscillates between two planes with angle of inclination θ. State its type of motion.
Is It
i) Periodic?
ii) Oscillatory?
iii) Non-SHM?



b) A tunnel is made along diameter of earth.
A particle is released from one end.
Is It
i) Periodic?
ii) Oscillatory?
iii) Non-SHM?

Ans.a) Because, acc is not proportional to displacement.

Acc. from AB $= g \sin\theta$

And AC $= g \sin\theta$ [but direction opposite]

Acc is not proportional to displacement. So, motion is periodic, oscillatory but not SHM.

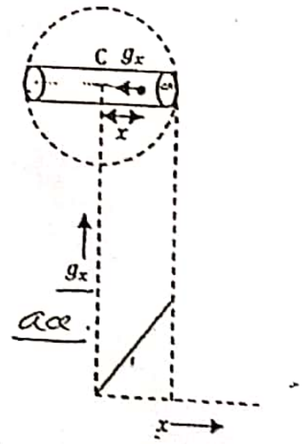
Note:

For a pendulum,

If angle is small → SHM
If angle is large → Non-SHM

b) As per theory of gravitation, acceleration is directly proportional to displacement i.e.

$$\boxed{g_x = \left(\frac{GM}{R^3}\right).x}$$



When particle is to right of centre C, acc. is towards left.
When particle is to left of centre C, acc. is towards right.

So, acc $= -\left(\frac{GM}{R^3}\right)$. displacement

i.e. acc $= -$ (constant).displacement

So, motion of ball inside the tunnel is S.H.M. Can you find the time period of the same. Try at the end of this chapter.

+1 / Ch 10 / Unit 10A / Q2 / Examples Of Oscillatory Motion

Q3. a) A particle moves in a circle in anti-clockwise direction. Plot displacement of shadow on y axis v/s time graph.

b) Define:

i) Time period ii) Frequency

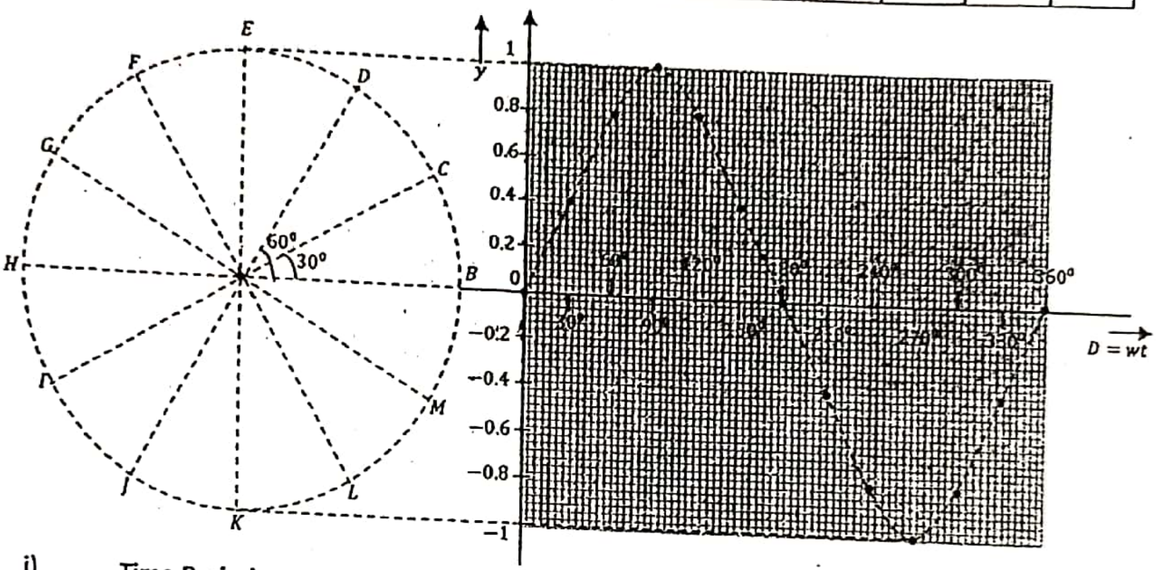
iii) Angular Frequency iv) Displacement

Explain using $w \to 30^0/\text{sec}$.

Ans.

a)

θ (wt)	0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0
y	0	0.5	0.85	1.0	0.85	0.5	0	-0.5	-0.85	-1.0	-0.85	-0.5	0



b)

i) **Time Period:**

It is the least interval of time after which the periodic motion of a body repeats itself.

In the given example, time period is 12 seconds i.e. 12 seconds to complete one round (360^0)

ii) **Frequency:**

It is define as the number of periodic motions executed by body per second.

In the given example, frequency is $\frac{1}{12}$

iii) **Angular Frequency:**

Angular frequency of a body executing periodic motion is equal to the product of frequency of the body with factor 2π.

Example: Angular frequency $= 2\pi \frac{1}{2} = \left[\frac{\pi}{6\,rad}\,\frac{}{sec}\right]$ i.e. $\left(\frac{30^0}{sec}\right)$

iv) **Displacement:**

It is the distance of the particle from the mean position at any instant of time.

Example: After 6 seconds, displacement = 2r (diameter)

displacement $\leftarrow$ $\boxed{y = A \sin wt}$

+1 / Ch 10 / Unit 10A / Q3 / Displacement Of Shadow SHM

Q4. Find expression for
a) Displacement
b) Velocity
c) Acceleration of a particle under SHM. Also plot graphs.

Ans. **a) Displacement:**

$$\boxed{y = A \sin \omega t} \quad \text{—— (1)}$$

b) Velocity:

$$v = \frac{d(y)}{dt}$$

$$= \frac{d(A \sin \omega t)}{dt}$$

$$= A\,\omega \cos \omega t \quad \left[\frac{d(\sin \omega t)}{dt}\right] = \cos \omega t.(\omega)$$

$$\boxed{v = A\omega \cos \omega t} \quad \text{—— (2)}$$

Here $(A\,\omega)$ = max. velocity (U_{max})

$$= A\,\omega\sqrt{1-\sin^2\omega t}$$

$$= A\,\omega\sqrt{1-\left(\frac{y}{A}\right)^2}$$

$$\boxed{v = \omega\sqrt{A^2-y^2}} \quad \text{—— (3)}$$

c) Acceleration:

$$acc. = \frac{d(v)}{dt}$$

$$= \frac{d(A\,\omega \cos \omega t)}{dt}$$

$$= A\,\omega(-\omega \sin \omega t)$$

$$\boxed{acc = -\omega^2(A \sin \omega t)} \quad \text{—— (4)}$$

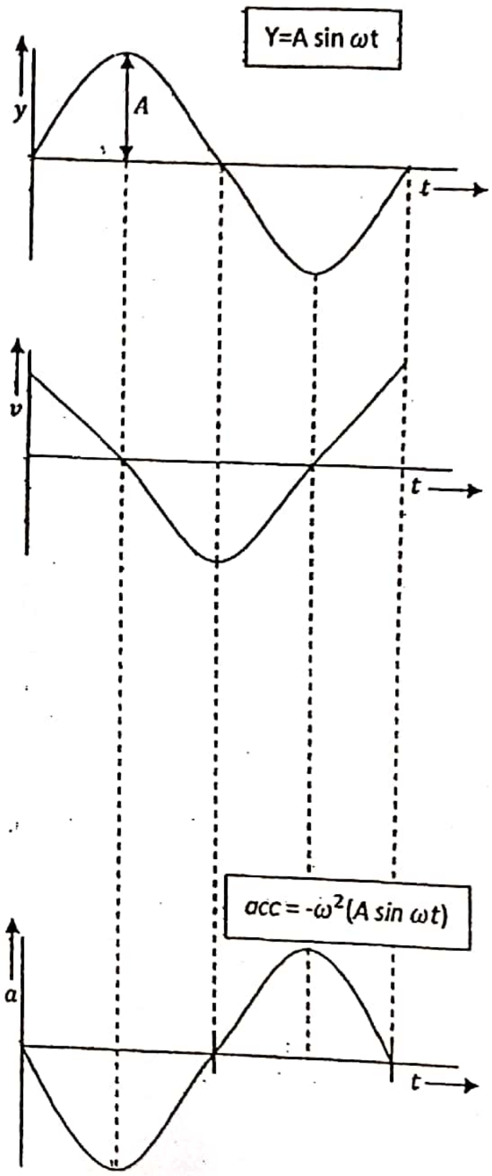
Here $(\omega^2 A)$ = max. acc. a_{max}

$$\boxed{a = -\omega^2 y} \quad \text{—— (5)}$$

i.e.

$$\boxed{acc = -(Const)\ displacement}$$

Condition for SHM

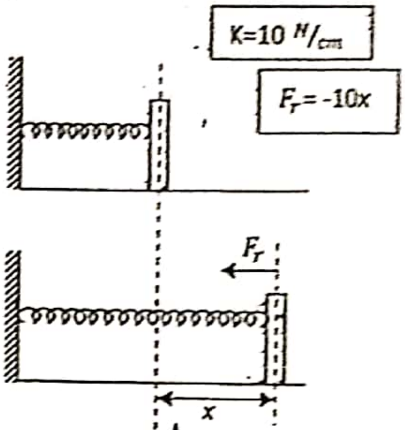


+1 / Ch 10 / Unit 10A / Q4 / Displacement Velocity And Acceleration Of Particle In SHM

Q5. a) **Prove that oscillations of spring mass arrangement is SHM. Find time period of the same.**

b) **Plot Potential Energy, Kinetic Energy, Total Energy graph for spring mass oscillation.**

Ans. a) **<u>Spring – Mass:</u>**



$F_{restor} \propto x$

$$\boxed{F_r = -k\,x}$$

$ma = -k\,x$

$$\boxed{a = -\left(\frac{k}{m}\right)x}$$

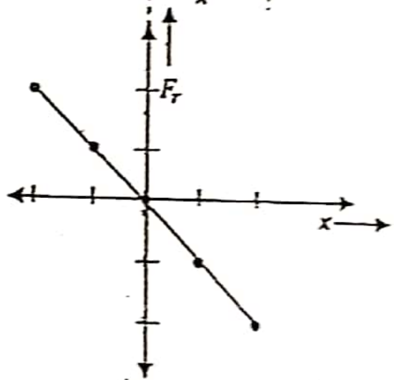
acc $= -\left(\frac{k}{m}\right)$ displacement ——— (1)

(Equation for SHM)

For SHM

acc $= -\omega^2$ (displacement) ——— (2)

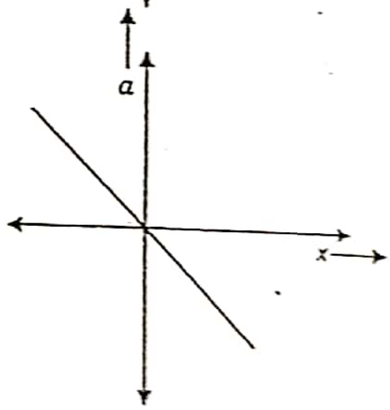
compare (1) and (2)



$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$



$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

b) i) **<u>Potential Energy:</u>**

$$\boxed{P.E_x = \frac{1}{2}kx^2}$$ ——— (1)

$P.E_t = \frac{1}{2}k\,(A\sin\omega t)^2$

$$\boxed{P.E_t = \left(\frac{1}{2}kA^2\right)\sin^2\omega t}$$ ——— (2)

max. value 1

$P.E_{max} = \frac{1}{2}kA^2$

+1 / Ch 10 / Unit 10A / Q5 / Time Period Of Spring Mass Arrangement

ii) **Kinetic Energy:**

$$K.E_x = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m \left[\omega\sqrt{A^2 - x^2}\right]^2$$

$$K.E_x = \frac{1}{2} m\, \omega^2 \,(A^2 - x^2)$$

$$= \frac{1}{2} \cancel{m} \frac{k}{\cancel{m}} (A^2 - x^2)$$

$$\boxed{K.E_x = \frac{1}{2} k\,(A^2 - x^2)} \tag{3}$$

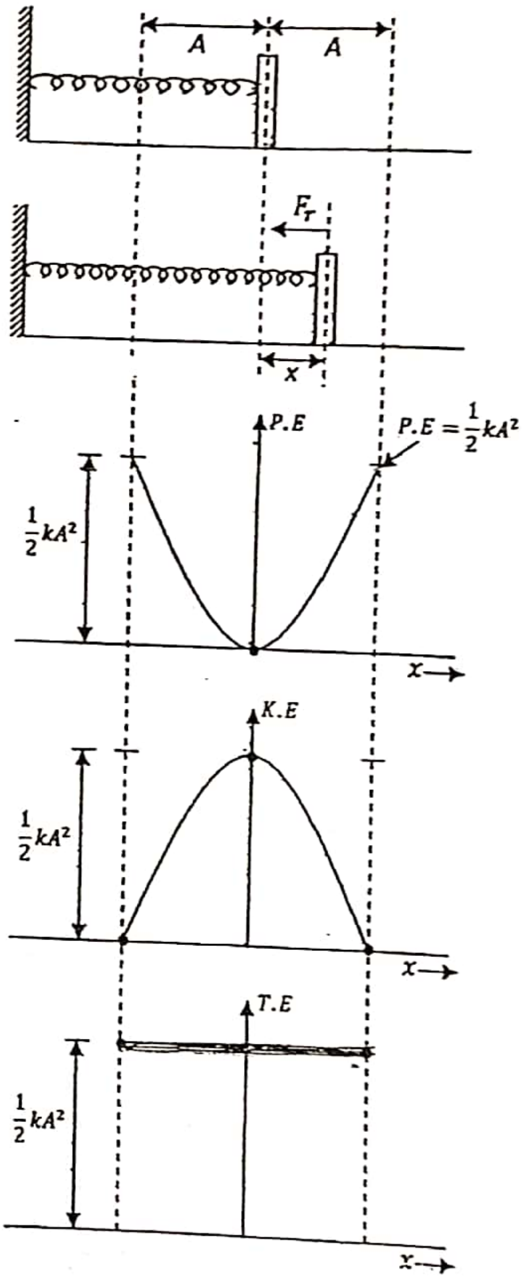
$$\boxed{\begin{array}{l} K.E_{max} = \frac{1}{2} kA^2, \text{ when } x = 0 \\ K.E_{min} = 0, \text{ when } x = \pm A \end{array}}$$

$$K.E_t = \frac{1}{2} k\,(A^2 - [A \sin \omega t]^2)$$

$$= \frac{1}{2} kA^2 - kA^2 \ \sin^2 \omega t$$

$$= \frac{1}{2} kA^2 \,(1 - \sin^2 \omega t)$$

$$\boxed{K.E_t = \frac{1}{2} kA^2 \cos^2 \omega t} \tag{4}$$



iii) **Total Energy:**

$$T.E_x = P.E_x + K.E_x$$

$$= \frac{1}{2} kx^2 + \frac{1}{2} k\,(A^2 - x^2)$$

$$\boxed{T.E_x = \frac{1}{2} kA^2}$$

$$T.E_t = P.E_t + K.E_t \tag{5}$$

$$= \frac{1}{2} kA^2 \ \sin^2 \omega t + \frac{1}{2} kA^2 \ \cos^2 \omega t$$

$$= \frac{1}{2} kA^2 \ (\sin^2 \omega t + \cos^2 \omega t)$$

$$\boxed{T.E_t = \frac{1}{2} kA^2} \tag{6}$$

Total Energy remains same.
It is independent of time (t) and displacement (x).

+1 / Ch 10 / Unit 10A / Q6 / Springs In Parallel And In Series

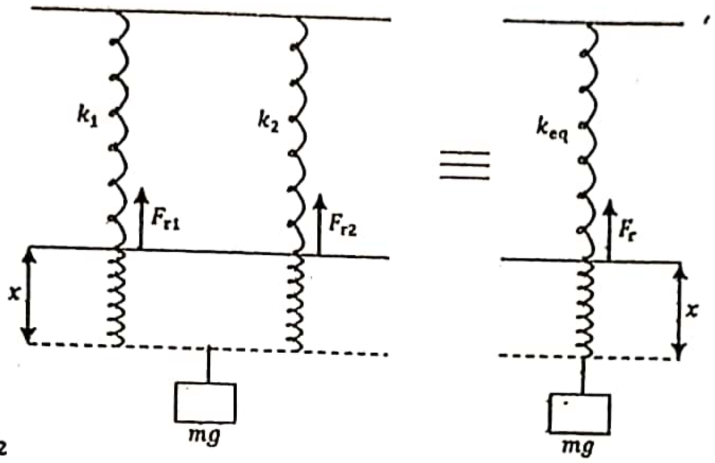
Q6. Find K_{eq} for

a) Two springs in parallel

b) Two springs in series

c) Mass sandwiched spring

Ans.a) <u>Springs in Parallel:</u>



mg $= F_{r1} + F_{r2}$

$$\boxed{mg = k_1 x + k_2 x} \quad \text{——— (1)}$$

for equivalent spring

mg $= F_r$

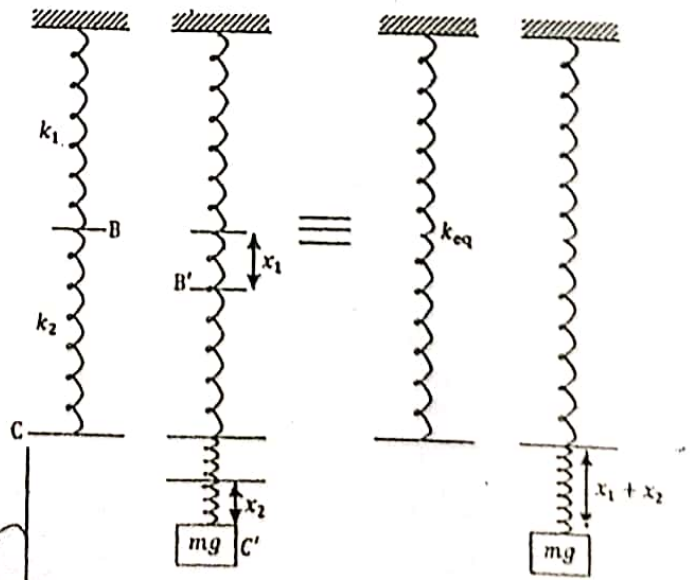
$$\boxed{mg = k_{eq}\, x} \quad \text{——— (2)}$$

from (1) and (2)

$k_{eq} \cancel{x} = k_1 \cancel{x} + k_2 \cancel{x}$

$$\boxed{k_{eq} = k_1 + k_2}$$

b) <u>Springs in Series:</u>

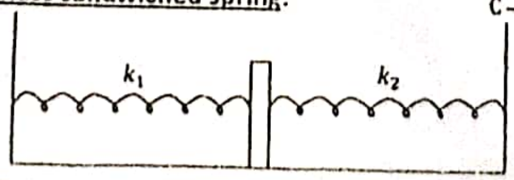


$x = x_1 + x_2$ $[F_r = k.x]$

$\frac{F_r}{k_{eq}} = \frac{F_r}{k_1} + \frac{F_r}{k_2}$

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$$

c) <u>Mass sandwiched spring:</u>

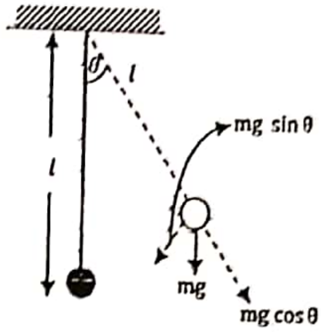


$$\boxed{k_{eq} = k_1 + k_2}$$

Q7. **Prove oscillations of a pendulum are SHM. Find its Time Period.**

Ans. **Step 1:**

Displace the particle from its mean position.



Step 2:

Check:→ If particle has tendency to come back to mean position, its oscillatory.

Step 3:

Find restoring force *OR* restoring torque. mg sinθ tries to bring the particle back to mean position.

$\tau_{res} = -(mg\sin\theta)\, l$ ——— (1)

$I\alpha = -mg\sin\theta\, l$

$m\, l^{\cancel{2}}\alpha = -mg\sin\theta\,\cancel{l}$

$\boxed{\alpha = -\left(\frac{g}{l}\right).\sin\theta}$ Not SHM ——— (2)

Step 4:

Apply limits

$\theta \to$ small $\qquad \sin\theta \simeq \theta$

$\boxed{\alpha = -\left(\frac{g}{l}\right)\theta}$ ——— (3)

angular acc = -(const) displacement
So, it is SHM

Step 5:

Compare with standard SHM equation

$\boxed{acc = -\omega^2 \text{ (displacement)}}$ ——— (4)

$\omega^2 = \frac{g}{l}$

$\omega = \sqrt{g/l}$

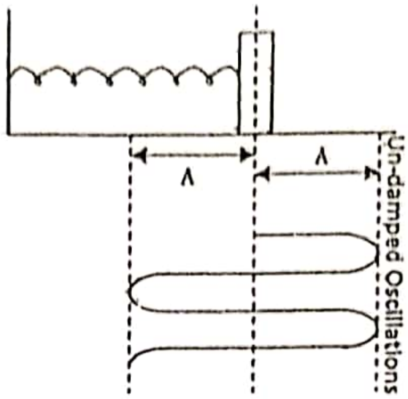
$\frac{2\pi}{T} = \sqrt{g/l}$

$\boxed{T = 2\pi\sqrt{l/g}}$

+1 / Ch 10 / Unit 10A / Q7 / Time Period Of Pendulum

Q8. a) What is un-damped and damped oscillations.
b) Free, forced and resonant oscillations.

Ans.a) **Un-damped Oscillations:**

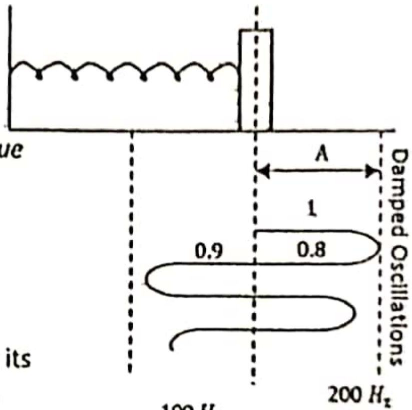


'Oscillation in which amplitude remains constant with time'.

Ideal spring-mass arrangement shown n figure.
Oscillations with constant amplitude A

$$F = -kx$$

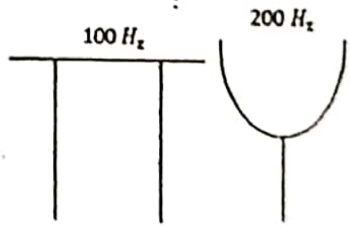
Damped Oscillation:



'Oscillation in which amplitude changes (decreases) due to dissipative force (like friction)'.

$$F = -kx - bv$$

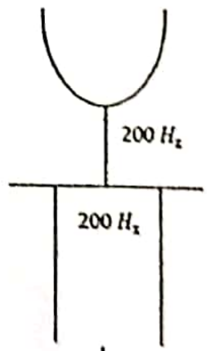
b) i) **Free Oscillation:**



Oscillation in which an object oscillates with its natural frequency is called Free Oscillation.

Example: A table oscillates with $100H_z$. Tuning fork oscillates with frequency $200H_z$

ii) **Forced Oscillation:**



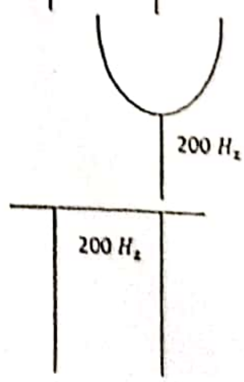
The oscillation in which the driver forces the driven to oscillates with its frequencies but not with its natural frequency.

Example: When a vibrating tuning fork is kept in contact with the table, the table also oscillates with same frequency as that of fork. Table is under the forced oscillation by the driver i.e. fork.

iii) **Resonant Oscillation:**

Oscillation in which natural frequency of both the driver and driven is same. Amplitude is high in this case.

Example: Both table and tuning fork oscillate with $200H_z$



+1 / Ch 10 / Unit 10A / Q8 / Damped Oscillation Free Forced And Resonant Oscillation

Q9. Find time period of Liquid in U-Tube.

Ans.

F_{res} = wt of liquid 2y

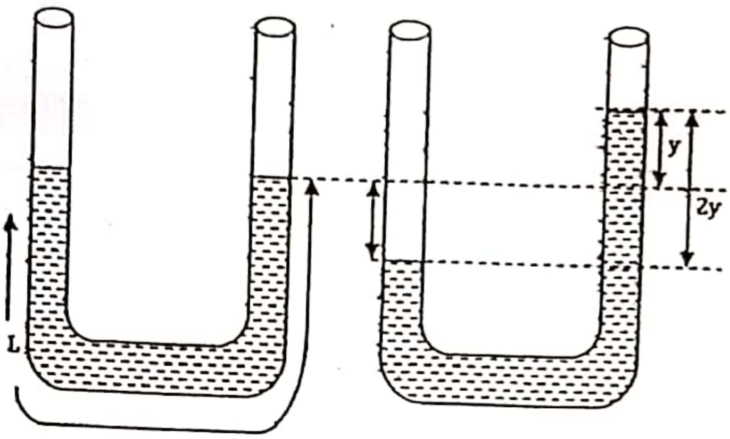
$= -(A\ 2y)\ \rho\ g$

Mass(a)= $-(2A\ \rho\ g)\ y$

$a = -\left(\frac{2A\ \rho\ g}{mass}\right) y$

$= -\left(\frac{2\cancel{A}\cancel{\rho}\, g}{\cancel{A}L\cancel{\rho}}\right) y$

$\boxed{acc = -\left(\frac{2g}{L}\right) y}$



Compare it with standard equation

acc $= -(\omega^2)$ displacement

$\omega^2 = \frac{2g}{L}$

$\omega = \sqrt{\frac{2g}{L}}$

$\boxed{T = 2\pi \sqrt{\frac{L/2}{g}}}$

+1 / Ch 10 / Unit 10A / Q9 /
Time Period Liquid In U-Tube

Q10. Find time period of Floating object.

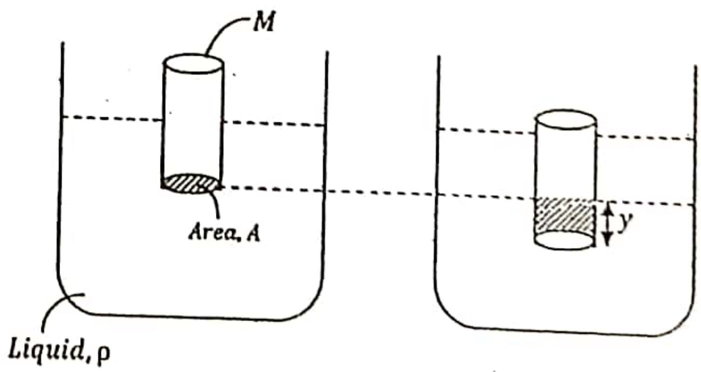
Ans.

F_{res} = extra up thrust

$= -(A\ y\ \rho)\ g$

Mass(a)= $-(A\ \rho\ g)\ y$

$a = -\left(\frac{A\ \rho\ g}{m}\right) y$



compare it with standard equation

acc $= -(\omega^2)$ displacement

$\boxed{T = 2\pi \sqrt{\frac{m}{A\ \rho\ g}}}$

++1 / Ch 10 / Unit 10A / Q10 /
Time Period Of Floating Object